EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng. Struct. Dyn. **27**, 439–463 (1998)

SINGLE- AND MULTIPLE-TUNED LIQUID COLUMN DAMPERS FOR SEISMIC APPLICATIONS

FAHIM SADEK[1], BIJAN MOHRAZ[2,*] AND H. S. LEW[3]

[1] *Southern Methodist University, Dallas, TX 75275, U.S.A., on assignment, Structures Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, U.S.A.*
[2] *Southern Methodist University, Dallas, TX 75275, U.S.A., on leave, Structures Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, U.S.A.*
[3] *Structures Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, U.S.A.*

SUMMARY

Design parameters for single- and multiple-tuned liquid column dampers for reducing the response of structures to seismic excitations are presented. A deterministic analysis is carried out using 72 earthquake ground motion records to determine the tuning ratio, tube width to liquid length ratio, and head loss coefficient corresponding to a given mass ratio for single-tuned liquid column dampers. A similar analysis is performed to determine the central tuning ratio, tuning bandwidth, and grouping of dampers for multiple-tuned liquid column dampers. The study indicates that by properly selecting the design parameters, single- and multiple-tuned liquid column dampers can reduce the response of structures to seismic excitation by up to 45 per cent. Design examples using single- and multiple-tuned liquid column dampers in a bridge and a ten-storey building are presented to illustrate how the parameters are selected and to demonstrate the performance of the devices under different ground excitations. The response of several structures with tuned liquid column dampers is compared with that using tuned mass dampers where it is shown that both devices result in comparable reductions in the response.

KEY WORDS: earthquake loading; energy dissipation; passive control; tuned liquid column dampers; tuned mass dampers

INTRODUCTION

Tuned liquid dampers (TLD) and tuned liquid column dampers (TLCD) are passive energy-absorbing devices that have been suggested for controlling vibrations of structures under different dynamic loading conditions. TLDs consist of rigid tanks filled with shallow liquid where the sloshing motion absorbs the energy and dissipates it by the viscous action of the liquid, wave breaking, and auxiliary damping appurtenances such as baffles, nets or floating beads. TLCDs consist of tube-like containers filled with liquid where energy is dissipated by the movement of the liquid through an orifice. Both devices have proved effective in reducing the response of structures to wind excitations[1–4] and have been installed in several structures. Examples include the 149·4 m-high Shin Yokohama Prince Hotel in Japan[4] with 30 TLDs placed on the top floor and the Higashi-Kobe cable-stayed bridge in Japan[5] with TLCD units attached to the bridge deck. For seismic applications, however, sufficient studies have not been carried out to assess the effectiveness of these devices in reducing the response.

* Correspondence to: Bijan Mohraz, Building and Fire Research Laboratory, Bldg 226, Room B-158, United States Department of Commerce, National Institute of Standards and Technology, Gaithersburg, MD 20899-0001, U.S.A.

CCC 0098–8847/98/050439–25$17·50
Received 26 February 1997
Revised 26 September 1997

TLCDs are relatively easy to install in new and existing buildings. They do not interfere with vertical and horizontal load paths as other passive devices may do. It is easy to adjust their frequencies as explained later, and they can be combined with active control devices[4,6,7] to function as hybrid systems. Unlike tuned mass dampers, TLCDs do not require a large space for stroke lengths. Furthermore, as Kareem[4] has demonstrated, TLCDs can be used to dissipate energy in two directions simultaneously by using a bi-directional U-tube. The configuration consists of partitioning the liquid container with a block that results in stacked U-tubes in both directions with a common base.

In this study, the effectiveness of single- and multiple-tuned liquid column dampers (STLCD and MTLCD) for seismic applications is examined. The response of several single-degree-of-freedom structures with TLCDs to 72 earthquake accelerograms is computed and used to select the design parameters (tuning ratio, tube width to liquid length ratio, and head loss coefficient) for STLCDs and (central tuning ratio, tuning bandwidth, and grouping of dampers) for MTLCDs. Two design examples—a bridge grider modelled as a single-degree-of-freedom structure and a ten-storey building modelled as a multi-degree-of-freedom structure; each equipped with STLCD and MTLCD—are used to illustrate the selection of the design parameters and their effectiveness in reducing the response to earthquake loading.

ANALYSIS

A tuned liquid column damper attached to a SDOF system is shown in Figure 1. The equation of motion of the liquid column is (see Reference 8):

$$\rho AL\ddot{y} + \tfrac{1}{2}\rho A\delta|\dot{y}|\dot{y} + 2\rho Agy = -\rho AB\ddot{x} \tag{1}$$

where A, B, L, ρ, and g are the cross-sectional area of the tube, tube width, liquid column length, liquid density, and acceleration due to gravity, respectively. The head loss coefficient δ depends on the orifice opening ratio (area of opening to cross-sectional area of tube) where $\delta = 0$ corresponds to full orifice opening and $\delta = \infty$ signifies full orifice closure. The value of δ in terms of the orifice opening ratio can be found in Blevins[9] or from experiments for specific orifice shapes and sizes. In the above equation, y represents

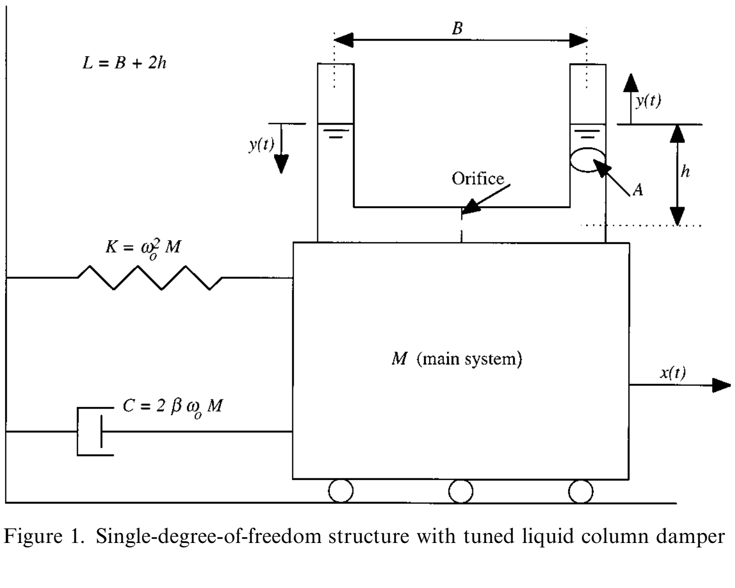


Figure 1. Single-degree-of-freedom structure with tuned liquid column damper

the elevation change of the liquid column and x the horizontal movement of the tube which is the same as that of the structure.

Recalling the equation of motion of a tuned mass damper (equation of a SDOF system) subjected to ground acceleration $\ddot{x}_g$ given by

$$\ddot{z} + 2\xi\omega_t\dot{z} + \omega_t^2 z = -\ddot{x}_g \tag{2}$$

where ξ is the damping ratio and ω_t the natural frequency of the TMD and comparing it with equation (1), it can be shown that a tuned liquid column damper can be considered as a tuned mass damper with a natural frequency ω_t given by

$$\omega_t = \sqrt{\frac{2g}{L}} \tag{3}$$

and a velocity-dependent damping ratio ξ expressed as

$$\xi = \frac{\delta}{4\sqrt{2gL}}|\dot{y}| \tag{4}$$

For a SDOF structure with mass M, natural frequency ω_0, damping ratio β and an attached TLCD (Figure 1), the equations of motion of the system subjected to ground acceleration $\ddot{x}_g$ are

$$\begin{bmatrix} M+\rho AL & \rho A\alpha L \\ \rho A\alpha L & \rho AL \end{bmatrix}\begin{Bmatrix}\ddot{x}\\ \ddot{y}\end{Bmatrix} + \begin{bmatrix} 2M\omega_0\beta & 0 \\ 0 & \frac{1}{2}\rho A\delta|\dot{y}| \end{bmatrix}\begin{Bmatrix}\dot{x}\\ \dot{y}\end{Bmatrix} + \begin{bmatrix} M\omega_0^2 & 0 \\ 0 & 2\rho Ag \end{bmatrix}\begin{Bmatrix}x\\ y\end{Bmatrix} = -\begin{Bmatrix} M+\rho AL \\ \rho A\alpha L\end{Bmatrix}\ddot{x}_g \tag{5}$$

where $\alpha = B/L$ is the ratio of the tube width to the liquid length. The presence of the term $|\dot{y}|$ in equations (1) and (5) indicates that TLCDs have a non-linear behaviour. Kwok *et al.*[1] and Xu *et al.*[2] used the method of equivalent linearization to solve the non-linear equations. They used a stochastic procedure to compute an equivalent damping coefficient c_p by minimizing the difference in the response between equation (1) and the equation of a SDOF system with a damping coefficient c_p. The equivalent damping is

$$c_p = 2\rho A\frac{\sigma_{\dot{y}}\delta}{\sqrt{2\pi}} \tag{6}$$

where $\sigma_{\dot{y}}$ is the standard deviation of the liquid velocity $\dot{y}$. Kwok *et al.*[1] and Xu *et al.*[2] computed the mean square response of structures with TLCDs to a zero-mean stationary Gaussian wind excitation. Since $\sigma_{\dot{y}}$ is not known *a priori*, they used an iterative procedure to solve the equations. In a later study, Sun[10] used the same linearization technique to arrive at the mean square response of a SDOF structure with a TLCD to a zero-mean stationary Gaussian ground acceleration to examine the effectiveness of the device for seismic applications. Instead of using an iterative procedure, Sun proposed approximate equations to compute the response. More recently, Won *et al.*[11] used the equivalent linearization technique to study the influence of the design parameters of TLCDs on the stochastic seismic response of structures. In their study, a time domain random vibration analysis was used where the ground motion was represented as a non-stationary random excitation with frequency and amplitude modulation.

For a deterministic analysis using digitized earthquake accelerograms, the equivalent linearization method cannot be used to solve equations (5) because of the non-linear characteristics. An iterative procedure is, therefore, used herein to compute the response. The method consists of estimating the liquid velocity at each time increment by using the first three terms in a Taylor series expansion of $\dot{y}$, i.e.

$$\dot{y}_{est}(t) = \tfrac{5}{2}\,\dot{y}(t-\Delta t) - 2\dot{y}(t-2\Delta t) + \tfrac{1}{2}\,\dot{y}(t-3\Delta t) \tag{7}$$

Using the estimated value of $\dot{y}$, the damping term $\rho A\delta|\dot{y}|/2$ in equation (5) and consequently the response (x, y, and their derivatives) of the structure are computed by solving equation (5). The difference between the estimated $\dot{y}_{est}$ and computed $\dot{y}_{com}$ is then calculated. If the relative error $|(\dot{y}_{est}-\dot{y}_{com})/\dot{y}_{com}|$ is greater than a desired tolerance (10^{-6} in this study), the procedure is repeated using $\dot{y}_{est}=\dot{y}_{com}$ until convergence is achieved. The method was examined for different accelerograms and it was found that usually one to three iterations were sufficient to achieve convergence.

DESIGN PARAMETERS FOR SINGLE-TUNED LIQUID COLUMN DAMPERS

The term single-tuned liquid column damper (STLCD) refers to one or more U-tubes with identical parameters. In addition to α and δ, the other STLCD parameters may be defined in terms of the tuning ratio f and mass ratio μ as

$$f=\frac{\omega_t}{\omega_0}=\frac{\sqrt{\frac{2g}{L}}}{\omega_0} \tag{8}$$

and

$$\mu=\frac{\rho AL}{M} \tag{9}$$

Similar to tuned mass dampers, in the design of tuned liquid column dampers the mass ratio is selected first and the remaining parameters are determined accordingly. For a given mass ratio μ, the parameters f, α, and δ were determined from the response of a number of SDOF structures with different f, α, and δ to a set of 72 horizontal components of accelerograms from 36 stations in the western United States (Appendix II). These accelerograms include a range of earthquake magnitudes (5·2–7·7), epicentral distances (6–127 km), peak horizontal ground accelerations (0·044–1·172g) and two soil conditions (rock and alluvium). The relative displacement and absolute acceleration response ratios were computed as the ratio of the peak response of the structure with a STLCD to the peak response without a STLCD. The design parameters were identified as those which result in the smallest mean response ratio.

Scaling of external excitation

It is noted from equation (1) that damping in a TLCD depends on the liquid velocity $\dot{y}$ and therefore, on the external excitation. Because earthquake records have different peak ground motions, they cannot be used on an absolute basis in a statistical analysis. In such cases, the records are scaled (normalized) to a common denominator (usually acceleration, velocity, or displacement) before they are used in the analysis. To determine whether ground velocity or acceleration‖ is a better scaling parameter, 30 SDOF structures with a 2 per cent damping and periods ranging from 0·1 to 3·0 s with increments of 0·1 s were analysed with and without STLCDs. The following typical STLCD parameters were considered: $\mu=0{\cdot}02$, $f=1$, $\alpha=0{\cdot}7$, and $\delta=0{\cdot}5$. The 72 records were scaled to a maximum ground acceleration $a=0{\cdot}25g$ and then to a maximum ground velocity $v=0{\cdot}30$ m/s. The coefficients of variation COV (the standard deviation divided by the mean) for the displacement and acceleration response ratios were computed and plotted in Figure 2. The figure shows that the COVs for records scaled to the same peak ground velocity are slightly smaller than those for records scaled to the same peak ground acceleration. The difference between the two COVs, however, is not that significant and since the ground acceleration is usually the most readily available ground motion data, it was used as the scaling parameter.

‖ Because of the errors inherent in baseline adjustment, displacements are seldom used as a scaling parameter

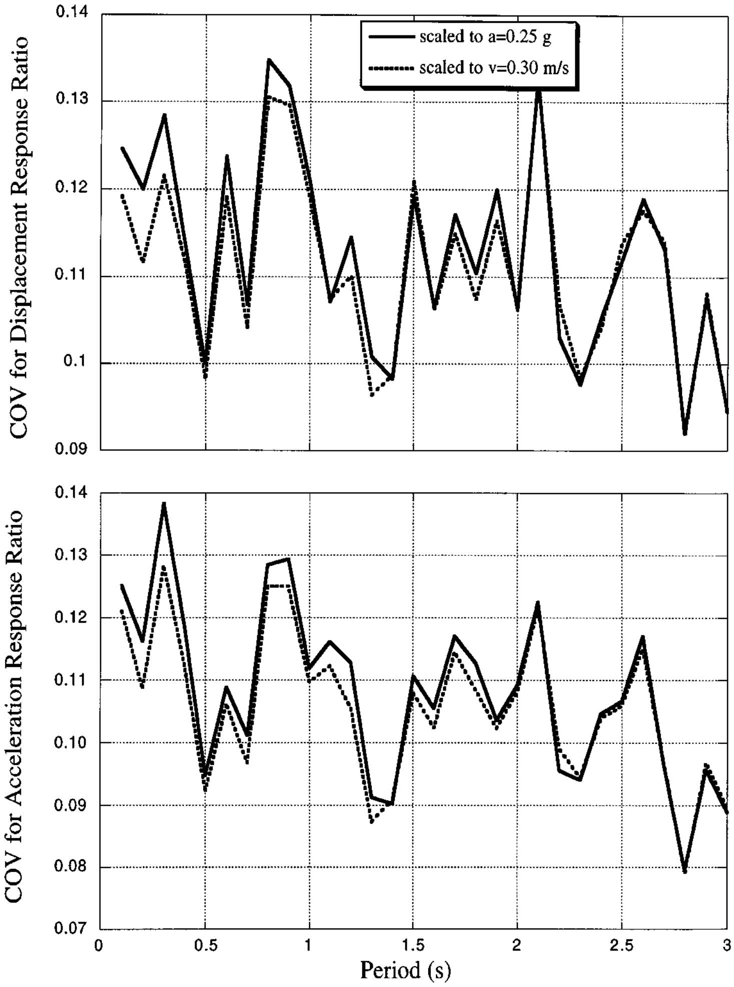


Figure 2. Coefficient of variation for response ratios for 2 per cent damped SDOF structures with STLCD with $\mu = 0{\cdot}01, f = 1{\cdot}0$, and $\delta = 0{\cdot}5$

Computation of design parameters

The design parameters (f_d, α_d, and δ_d) are determined for four mass ratios $\mu = 0{\cdot}005$, $0{\cdot}01$, $0{\cdot}02$ and $0{\cdot}04$ and two structural damping ratios $\beta = 0{\cdot}02$ and $0{\cdot}05$. These ratios include the range of practical interest for most structural applications. To obtain each design parameter, the influence of that parameter on the response of a SDOF system with a STLCD is examined by varying that parameter while keeping the other two constant. Similar to TMDs, the tuning and damping ratios of the damper are independent of the structure's period. Therefore, a period of 1.0 s was used for determining the parameters. The following discusses how the design parameters are determined:

Tuning ratio f. To select f, typical tube width to liquid length ratio $\alpha = 0{\cdot}7$ and head loss coefficient $\delta = 0{\cdot}5$ were considered. The accelerograms were scaled to a peak ground acceleration $a = 0{\cdot}25g$. Tuning ratios f ranging from $0{\cdot}8$ to $1{\cdot}2$ with increments of $0{\cdot}01$ were used in the analysis. The mean displacement and acceleration response ratios for the four mass ratios are shown in Figure 3 which show that for a STLCD, the higher the mass ratio the better its performance (lower response). Similar to tuned mass dampers, the tuning

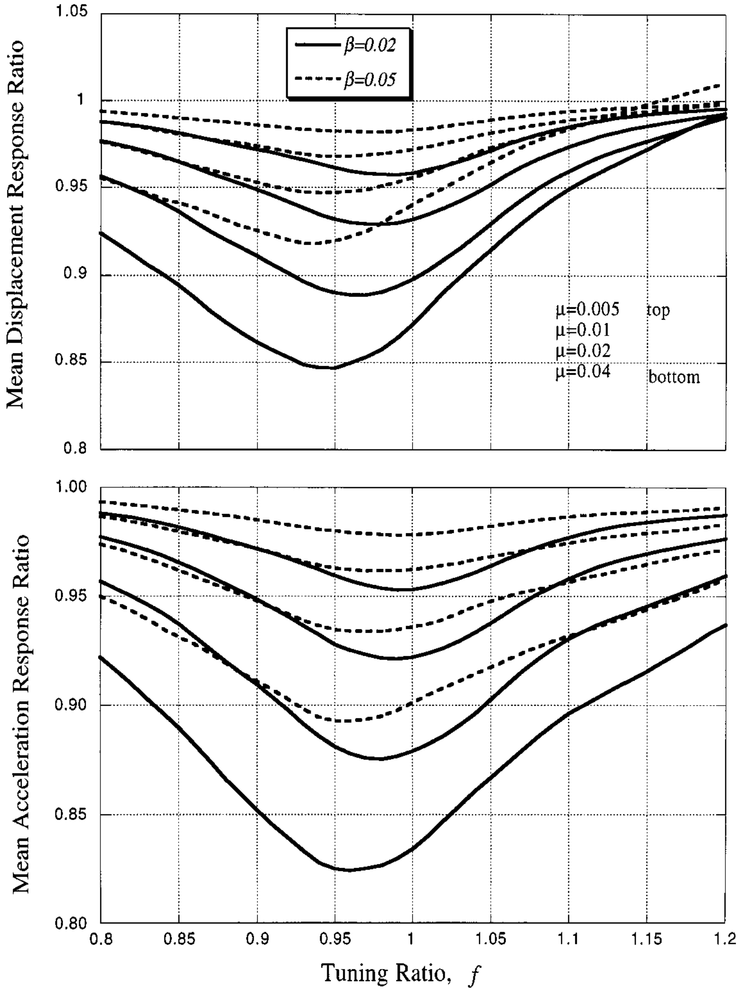


Figure 3. Variation of mean response ratios with tuning ratio f for different structural damping ratios and STLCD mass ratios

ratio depends not only on the mass ratio but also on the damping of the structure. Based on the results in Figure 3 and the range of mass and damping ratios considered in this study, the design tuning ratio f_d is found to be very close to the tuning ratio of TMDs for a white noise ground acceleration given by Warburton[12] as

$$f_d = \frac{\sqrt{1 - \frac{\mu}{2}}}{1 + \mu} \tag{10}$$

Equation (10) does not reflect the damping of the structure β. Tuning ratios for different structural damping coefficients may be obtained from Reference 12.

Tube width to liquid length ratio α. To select α, a head loss coefficient $\delta = 0{\cdot}5$ and the design tuning ratio f_d computed from equation (10) were considered. The 72 records scaled to a peak ground acceleration $a = 0{\cdot}25g$ were used in the analyses. Computations were carried out for values of α ranging from 0·1 to 0·9 with increments of 0·05. The results are presented in Figure 4 where it is observed that the larger the α, the larger the response reduction. Sun[10] reported that increasing α increases the root mean square displacement of the

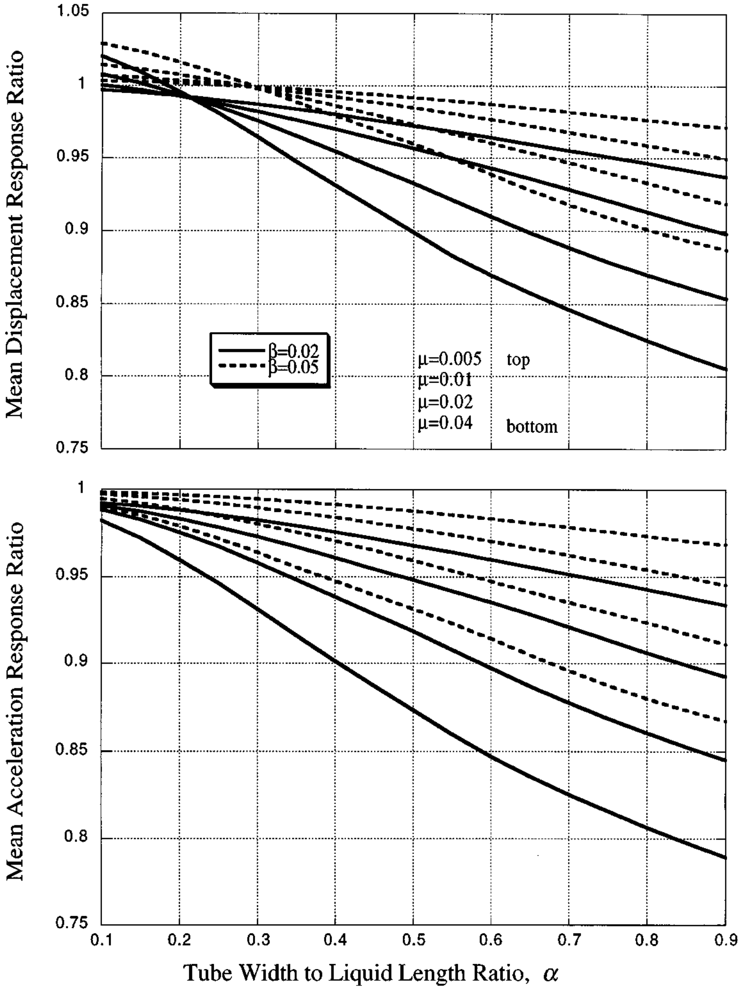


Figure 4. Variation of mean response ratios with α

structure. This study, however, suggests (see Figure 4) that α should be as large as possible as long as liquid is retained in the horizontal segment of the U-tube. If y_{max} is the anticipated maximum change in liquid elevation, then

$$\alpha = 1 - 2\frac{|y_{max}|}{L} \tag{11}$$

Since y_{max} depends on the excitation, an iterative procedure was used to determine α_d. Based on different analyses, it was determined that α_d should be between 0·75 and 0·80 for moderate to strong ground motions (accelerations up to $0{\cdot}7g$); $\alpha_d = 0{\cdot}8$ is used in this study.

The above discussion indicates that for best performance of TLCDs, it is preferable for the horizontal segment of the U-tube to have a much larger cross-sectional area than the vertical segments. In such a case, more energy is dissipated by the movement of the liquid in the large horizontal segment of the tube while the slender vertical segments act as a reservoir for the moving liquid.

Head loss coefficient δ. For a tuned mass damper, it is customary to define the damping ratio ξ as a function of the mass ratio μ and structural damping ratio β.[12–14] From the similarity between TMD and

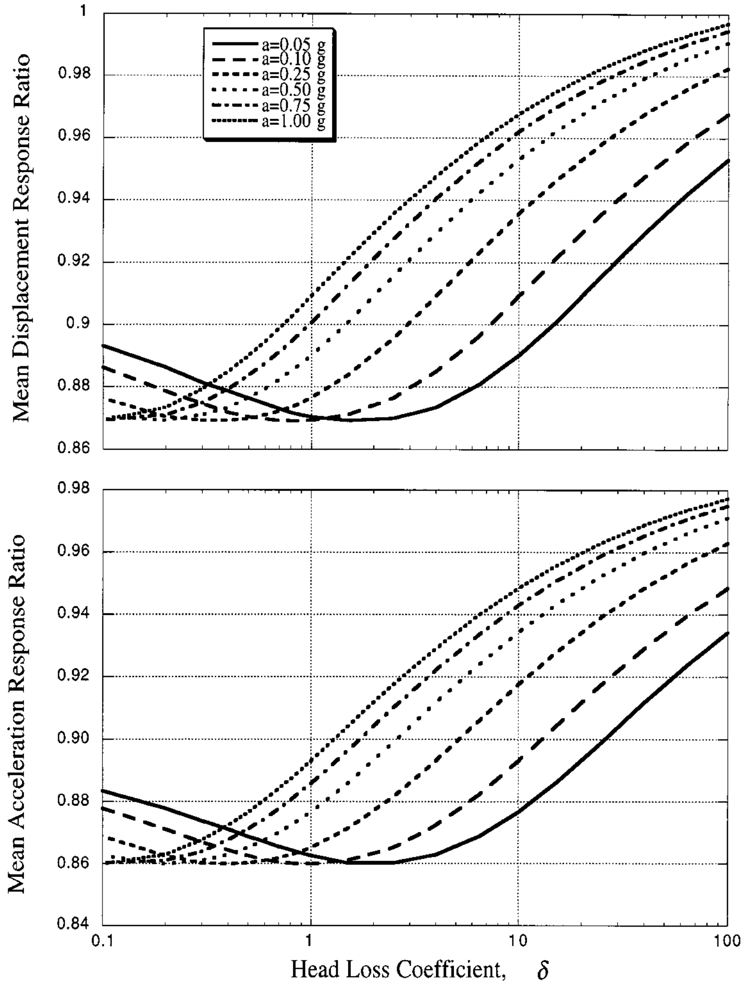


Figure 5. Variation of mean response ratios with δ

TLCD, it may be concluded from equation (4) that the head loss coefficient δ not only depends on β and μ but also on $\dot{y}$ or ground excitation. To determine δ_d, a SDOF structure ($T = 1{\cdot}0$ s, $\beta = 0{\cdot}02$) with a STLCD ($\mu = 0{\cdot}02$, $\alpha_d = 0{\cdot}8$, $f = f_d$) was subjected to the 72 earthquake accelerograms scaled to maximum accelerations of 0·05, 0·1, 0·25, 0·5, 0·75, and $1g$. The head loss coefficient δ was varied from 0·1 to 100 with four equally spaced intervals in each logarithmic cycle. The mean displacement and acceleration response ratios were computed and plotted in Figure 5 which show that the same reduction in the response can be obtained for different ground excitations by using an appropriate δ. Haroun *et al.*[7] have indicated that for the maximum displacement reduction, δ should be 0·4 and for the RMS displacement reduction, δ should be 0·8 regardless of the peak ground acceleration. This study, however, suggests (Figure 5) that for the maximum displacement reduction δ should vary according to ground acceleration.

The procedure was repeated with different mass and damping ratios and it was found that for a given mass and damping ratio, the product of δ_d and ground acceleration remains a constant. Thus,

$$\delta_d \left(\frac{a}{g} \right) = \eta \tag{12}$$

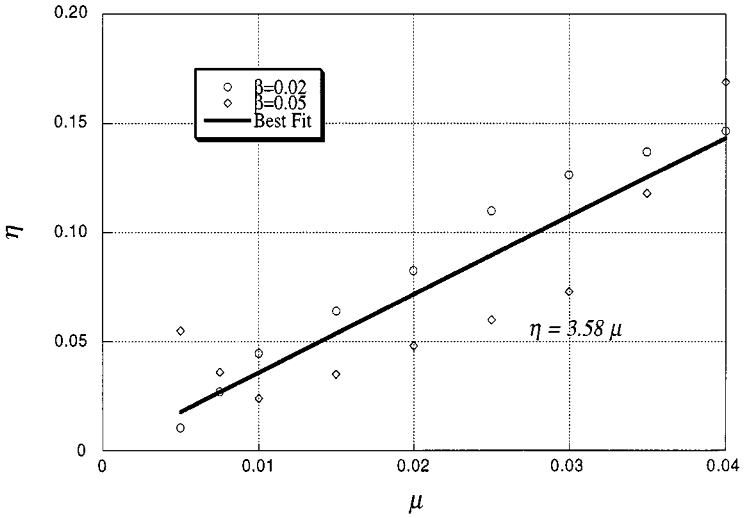


Figure 6. Relationship between constant η and mass ratio μ

where the constant η depends on the mass ratio μ and structural damping ratio β. For the range of mass and damping ratios considered herein, it was found that η depends more on the mass ratio than on the damping ratio as shown in Figure 6. A simple expression for estimating η in terms of μ using the best fit is given below:

$$\eta = 3{\cdot}58\mu \tag{13}$$

The head loss coefficient for the maximum displacement reduction can be obtained from equations (12) and (13) as

$$\delta_d = \frac{3{\cdot}58\mu}{a/g} \tag{14}$$

It should be noted that the above equation agrees with the observation made by Won *et al.*,[11] who conclude that the optimal head loss coefficient δ increases when the mass ratio μ increases and the load intensity decreases.

Selection of design parameters

From the previous analyses, the selection of design parameters for STLCDs may be summarized as follows: the mass ratio μ should be determined based on the trade-off between the desired reduction in the response and the cost, space, and weight of the dampers. Once the mass ratio is selected, the tuning ratio f_d and the head loss coefficient δ_d, which depends on the expected ground acceleration, can be determined from equations (10) and (14). The tuning ratio is used to find the liquid length L from equation (8) and the head loss coefficient is used to obtain the orifice opening.[9] Using $\alpha_d = 0{\cdot}8$ as suggested previously, the tube width B can be determined. For structures with large masses, it is practical to use a number of U-tubes to achieve the desired mass ratio. The cross-sectional area of the individual U-tubes is computed by dividing A, equation (9), by the number of tubes.

This method was used to select the STLCD parameters for SDOF structures with periods between 0·1 and 3·0 s with increments of 0·1 s, mass ratio of 0·005, 0·01, 0·02, and 0·04, and damping ratios of 0·02 and 0·05. The ground excitations included the 72 accelerograms scaled to a peak ground acceleration of 0·25g. The mean displacement and acceleration response ratios are shown in Figure 7 where it is observed that

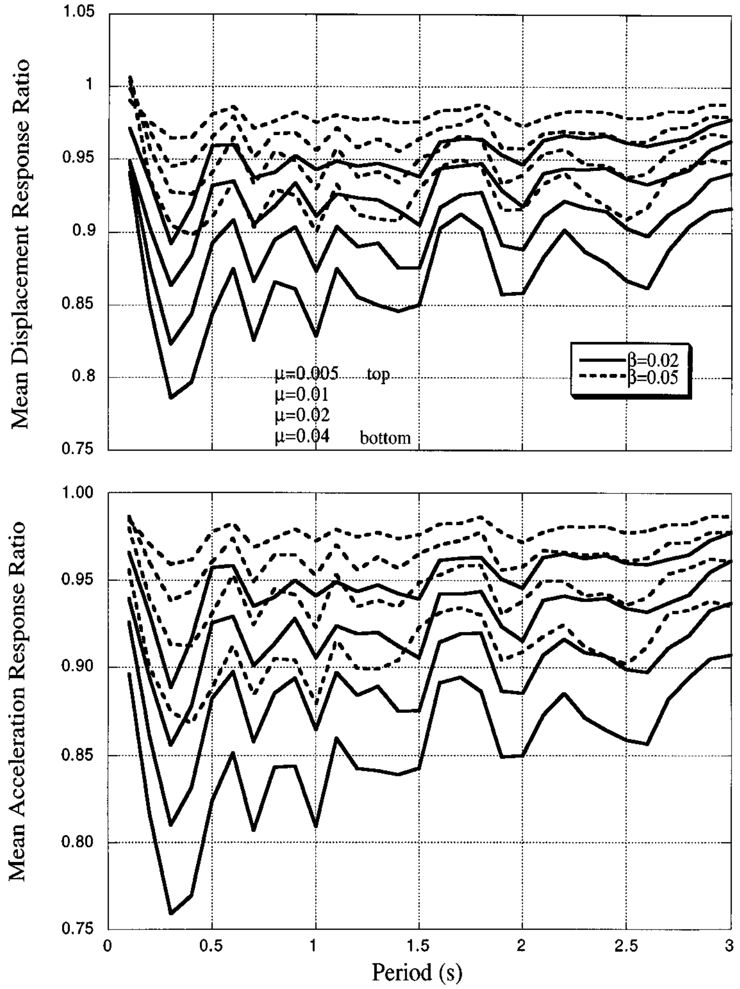


Figure 7. Mean response ratios for SDOF structures with STLCD

reductions in displacements and accelerations may be achieved with TLCDs, particularly for structures with small damping ratios. Increasing the mass ratio results in a higher damping in the system and, consequently, a better response reduction.

Comparison with tuned mass dampers

Sadek *et al.*[14] used an analysis similar to that presented in the previous section for SDOF structures with tuned mass dampers. A comparison of their results for TMDs and those for TLCDs is shown in Figure 8 for the 30 SDOF structures with periods ranging from 0·1 to 3·0 s with a damping ratio $\beta = 0{\cdot}02$. The responses with TMDs are normalized to those with TLCDs for two mass ratios $\mu = 0{\cdot}02$ and 0·04. The figure indicates that, for identical mass ratios, similar reductions in the response are obtained with both TLCD and TMD. TLCDs, however, have the following advantages over TMDs: (a) they do not require large stroke lengths; (b) it is easy to tune their frequency by adjusting their liquid column length L, see equation (3) and (c) if desired, they are capable of providing control in two directions simultaneously. On the other hand, the small density of water or other liquids in TLCDs relative to those of steel, concrete or lead in TMDs necessitates large spaces to produce the same mass ratio and thus the same damping effect.

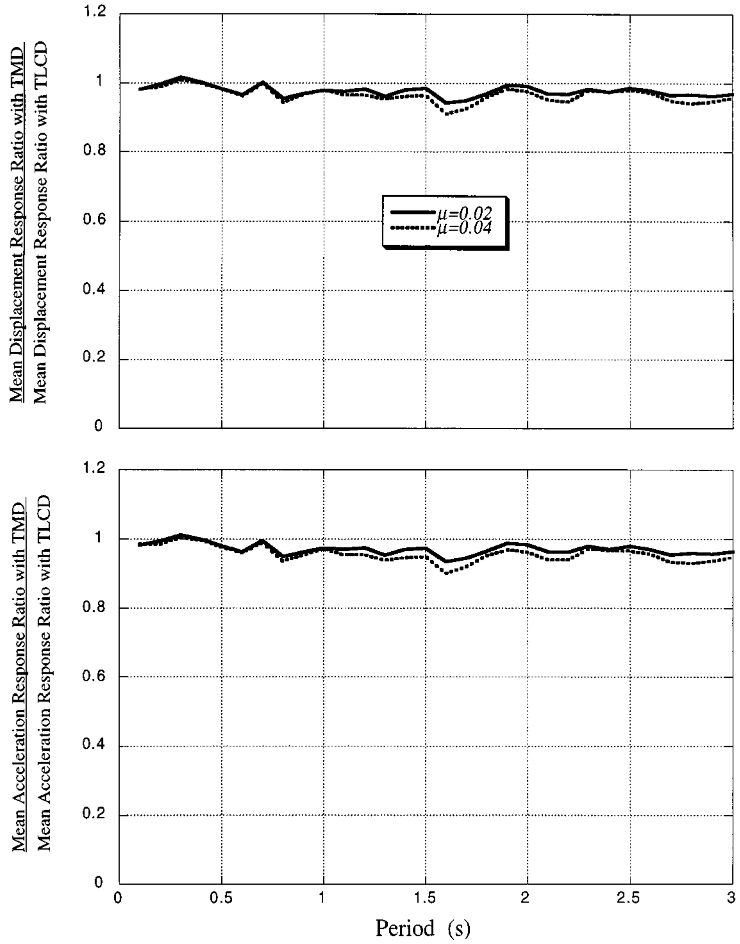


Figure 8. Comparison of mean displacement and acceleration response ratios for SDOF structures with TMD and TLCD for a structural damping ratio $\beta = 0{\cdot}02$

DESIGN PARAMETERS FOR MULTIPLE-TUNED LIQUID COLUMN DAMPERS

The term multiple-tuned liquid column damper (MTLCD) refers to two or more TLCD groups, each group with a different set of design parameters. The number of U-tubes in each group may range from one to several hundred in order to meet the required liquid mass. In this study, the same tube proportion α, cross-sectional area A, liquid density ρ, and head loss coefficient δ are used for each group. The only variable is the liquid length L which influences the tuning ratio f_i (see equation (8)). The difference between adjacent tuning ratios $(f_{i+1} - f_i)$, however, is assumed constant. Referring to Figure 9, the system may be characterized in terms of its central tuning ratio f_0, tuning bandwidth Δf, and the number of TLCD groups N, where

$$f_0 = \frac{f_N + f_1}{2} \tag{15}$$

and

$$\Delta f = \frac{f_N - f_1}{f_0} \tag{16}$$

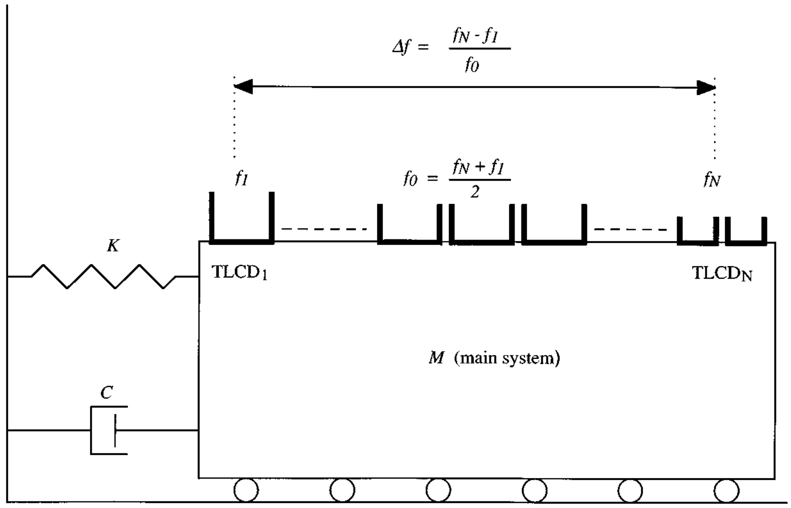


Figure 9. Single-degree-of-freedom structure with multiple-tuned liquid column damper

Once the tuning ratio f_i for each group is determined, the liquid length L_i can be computed from equation (8) and the tube cross-sectional area A from equation (9) by substituting $\sum L_i$ for L. Analyses were carried out for $\alpha_d = 0{\cdot}8$ and δ_d computed from equation (14) using the total mass ratio of all units. The mean response ratios were computed using the same 72 accelerograms scaled to a peak ground acceleration of $0{\cdot}25g$. Parametric studies were carried out to determine the influence of the parameters Δf, N, and f_0 on the MTLCD performance.

Tuning bandwidth Δf

A SDOF structure with an assumed period $T = 1{\cdot}0$ s and damping ratio $\beta = 0{\cdot}02$, with several TLCD groups (seven in this analysis) was considered in determining the influence of Δf. Mass ratios of 0·005, 0·01, 0·02, and 0·04 and a central tuning ratio $f_0 = 1{\cdot}0$ were assumed. The tuning bandwidth Δf was varied from 0 (STLCD) to 0·4 with increments of 0·02. The mean displacement and acceleration response ratios are shown in Figure 10 which indicate that a better reduction in response is obtained for a Δf other than zero. For maximum displacement reduction, Δf was found to be 0·125, 0·1, 0·05, and 0·025 for mass ratios of 0·04, 0·02, 0·01, and 0·005, respectively.

Number of TLCD groups N

To find the number of TLCD groups, the SDOF structure used previously was used with different number of TLCD groups. The tuning bandwidths from Figure 10 and a central tuning ratio $f_0 = 1{\cdot}0$ were used in the analysis. The number of TLCD groups N was varied from 1 (STLCD) to 31. The results are shown in Figure 11 which indicate that $N = 5$ is the most desirable number for reducing the response. The figure shows that for small mass ratios, there is no advantage in selecting MTLCDs over STLCDs.

Central tuning ratio f_0

To determine the central tuning ratio f_0, the same SDOF structure with five TLCD groups and the bandwidths Δf determined from Figure 10 was considered. The central tuning ratio f_0 was varied from 0·8 to 1·2 with increments of 0·1. The mean displacement and acceleration response ratios for different mass ratios μ computed using the 72 accelerograms are shown in Figure 12. The plots show that the most reduction in

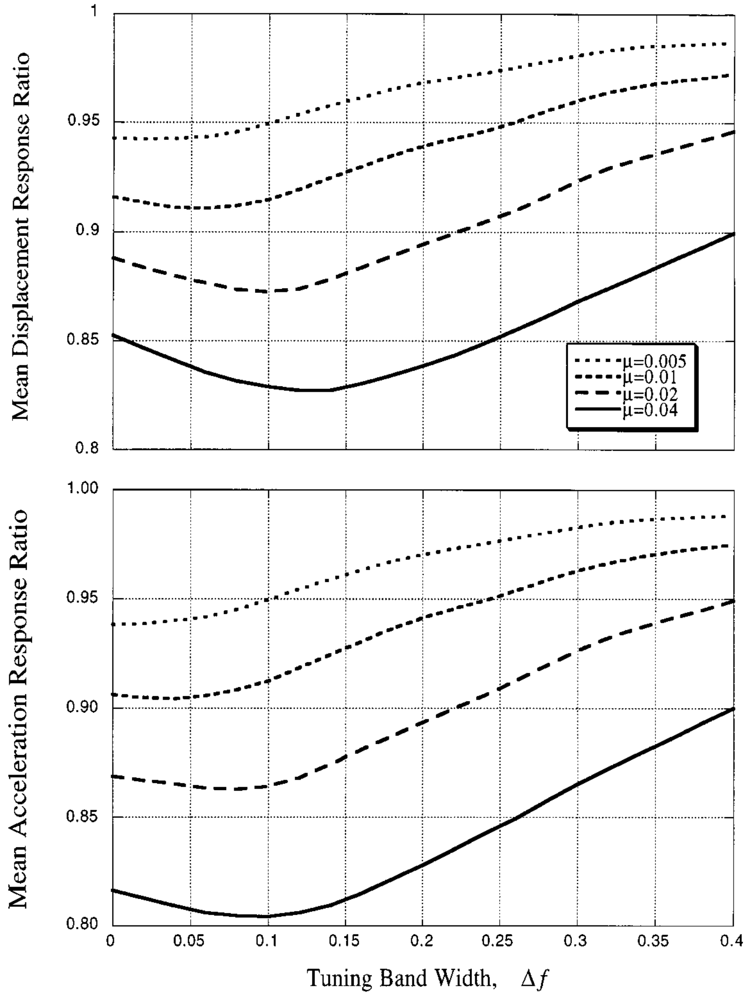


Figure 10. Variation of mean response ratios with Δf

the response is obtained for a central tuning ratio of approximately 1·0 indicating that for the best performance, the central frequency of a MTLCD should be tuned to the natural frequency of the structure.

Selection of design parameters

From the previous analyses, the selection of MTLCD parameters may be summarized as follows: after selecting the mass ratio μ, the head loss coefficient δ_d and consequently the orifice opening for all units can be determined similar to STLCDs. The mass ratio is also used to determine the tuning bandwidth Δf. This study indicates that Δf should be 0·125, 0·1, 0·05, and 0·025 for mass ratios $\mu = 0{\cdot}04$, 0·02, 0·01, and 0·005, respectively. Using five TLCD groups (each TLCD group may contain several U-tubes) with the central frequency tuned to that of the structure, the tuning ratio f_i for each group can be determined from equations (15) and (16) and the liquid lengths L_i from equation (8). Using $\alpha = 0{\cdot}8$, the tube width B_i can be determined. The U-tubes in each group should have the same cross-sectional area which is computed from the required liquid mass.

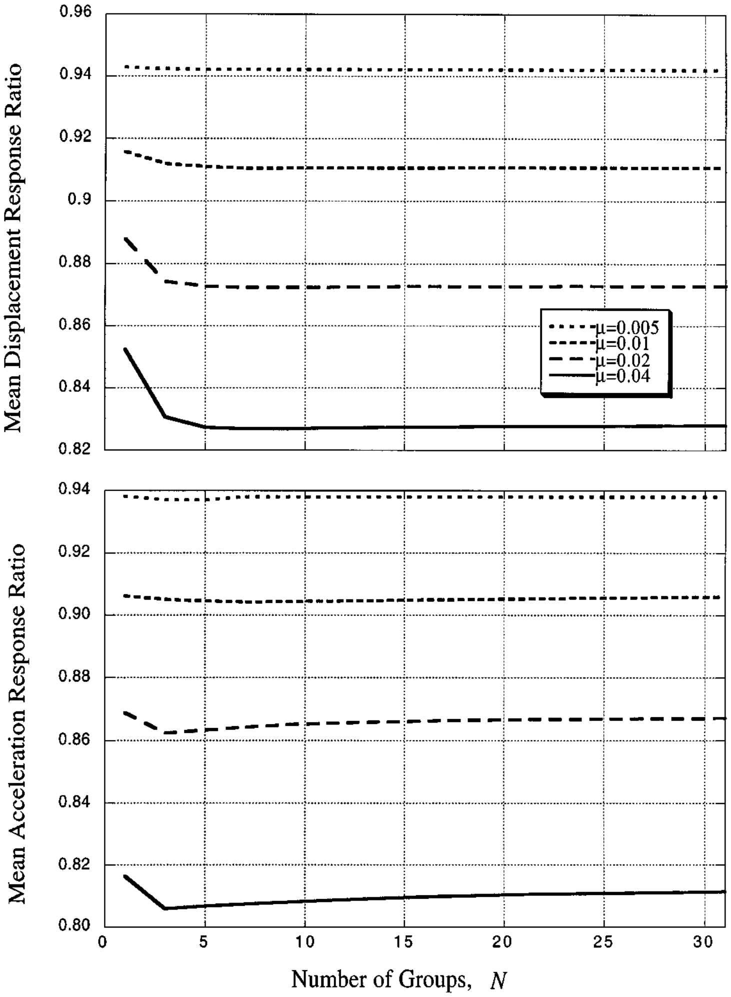


Figure 11. Variation of mean response ratios with N

The procedure was used to select the MTLCD parameters for SDOF structures with periods between 0·1 and 3·0 s with increments of 0·1 s, a damping ratio of 0·02, and mass ratios of 0·005, 0·01, 0·02, and 0·04. The mean displacement and acceleration response ratios of the structures to the 72 accelerograms scaled to peak ground acceleration 0·25g are shown in Figure 13.

Robustness of MTLCD

Comparing the mean displacement and acceleration response ratios for structures with MTLCDs (Figure 13) and STLCDs (Figure 7), practically no improvement in the displacement and acceleration responses is obtained. Similar observations have been reported by Yamaguchi and Harnpornchai[15] for multiple tuned mass dampers and by Fujino and Sun[3] for multiple tuned liquid dampers. In both studies, however, multiple TMDs and TLDs proved to be robust (less sensitive) to changes in structural parameters and external excitations.

To demonstrate the robustness of MTLCDs over STLCDs, a SDOF structure with 2 per cent damping with an assumed period of 1·0 s is selected. Suppose the actual stiffness of the structure corresponds to

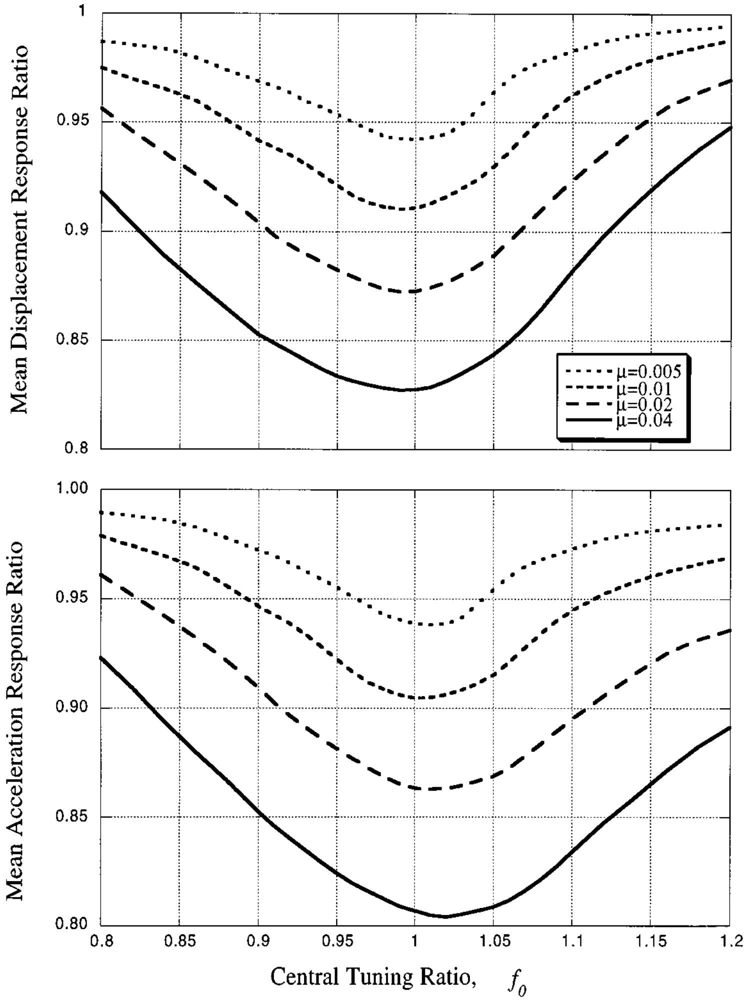


Figure 12. Variation of mean response ratios with f_0

a period other than 1·0 s, say for example 0·95 s. Using the assumed period of 1·0 s and a mass ratio of 0·04, the parameters for a STLCD and a MTLCD were selected. The analysis of the structure with STLCD and with MTLCD subjected to the S90W component of the El Centro accelerogram, the Imperial Valley earthquake, 1940, scaled to a maximum ground acceleration of 0·25g shows a maximum relative displacement and absolute acceleration of 99 mm and 0·44g for the structure with STLCD , and 89 mm and 0·39g for the structure with MTLCD; thus illustrating the robustness of MTLCDs over STLCDs.

EXAMPLES

Two examples are presented to show the selection of single-and multiple-tuned liquid column dampers and to demonstrate their performance under different seismic excitations.

Bridge modelled as SDOF structure

A simple-span box-girder concrete bridge is modelled as a SDOF system with a mass $M = 1 \times 10^6$ kg, natural period $T = 2$ s, and damping ratio $\beta = 0{\cdot}02$. The bridge is to be designed for ground acceleration of

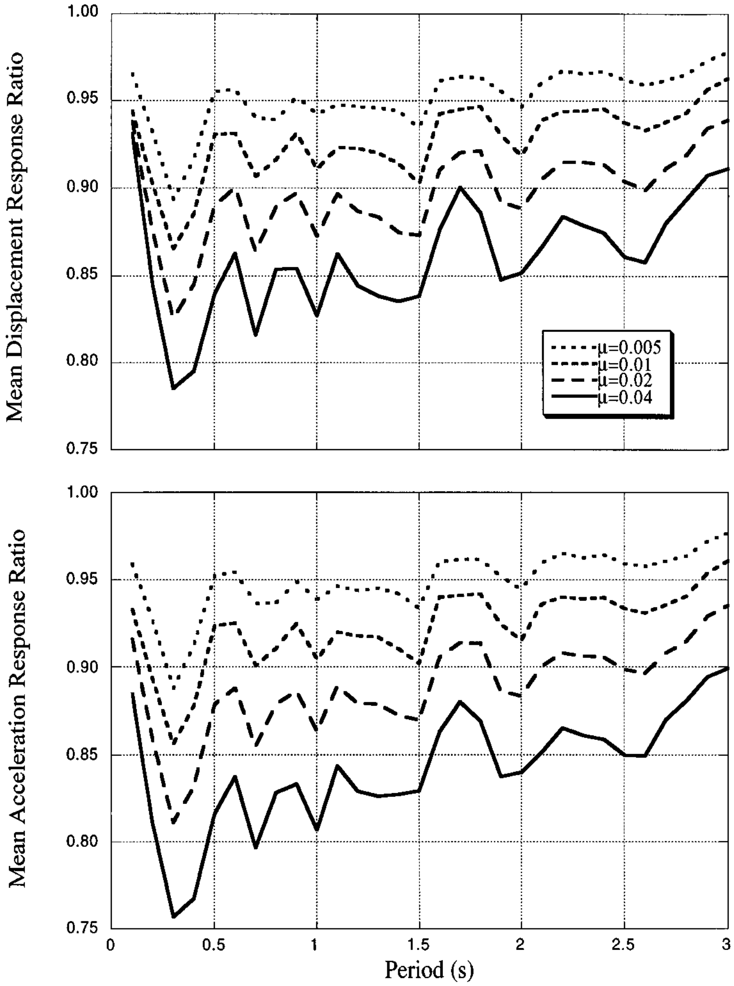


Figure 13. Mean response ratios for SDOF structures with MTLCD

0·25g. Tuned liquid column dampers are attached to the inside of the box girder to reduce the seismic response of the bridge. The mass ratio is assumed to be 0·04. The parameters of the STLCD are selected as follows: for the mass ratio of 0·04, the tuning ratio and the head loss coefficient are obtained from equations (10) and (14) as $f_d = 0{\cdot}952$ and $\delta_d = 0{\cdot}573$, respectively. The tuning ratio is used to compute the liquid length $L = 2{\cdot}2$ m from equation (8) and the head loss coefficient is used to find the orifice opening ratio as 0·75.[9] Using $\alpha_d = 0{\cdot}8$ as suggested previously, the tube width B will be 1·76 m. To achieve the required mass ratio, 600 U-tubes, each with a cross-sectional area of 0·03 m^2, filled with water may be used.

If one were to use MTLCD, five TLCD groups would be selected (Figure 11). Using $\Delta f = 0{\cdot}125$ and $f_0 = 1$ from Figures 10 and 12, respectively, the tuning ratios f_i from equations (15) and (16) for the five groups would be 1·065, 1·033, 1·0, 0·968, and 0·935. The corresponding liquid lengths L_i from equation (8) would be 1·75, 1·86, 1·99, 2·13 and 2·27 m. To achieve a mass ratio of 0·04 with water, a total of 100 U-tubes, each with a cross-sectional area of 0·04 m^2 should be used in each group. The orifice opening and the tube width to liquid length ratio are the same as those for the STLCD. The response of the bridge with no control, with STLCD, and with MTLCD to the following ground excitations: the S90E component of El Centro, the

Table I. Response of the bridge with and without TLCD

Control	El Centro, 1940 x_{max} m	El Centro, 1940 a_{max} g	Taft, 1952 x_{max} m	Taft, 1952 a_{max} g	Cholame, 1996 x_{max} m	Cholame, 1996 a_{max} g	Pacoima Dam, 1971 x_{max} m	Pacoima Dam, 1971 a_{max} g
None	0·364	0·367	0·147	0·148	0·333	0·336	0·059	0·059
STLCD	0·194	0·190	0·114	0·117	0·259	0·268	0·053	0·054
MTLCD	0·200	0·196	0·115	0·117	0·257	0·261	0·053	0·054

Imperial Valley earthquake, 1940; the S69E component of Taft Lincoln School Tunnel, Kern County earthquake, 1952; the N40W component of Cholame, Shandon, California Array #12, the Parkfield earthquake, 1966; and the S74W component of Pacoima Dam, the San Fernando Earthquake, 1971; all scaled to a peak ground acceleration of $0{\cdot}25g$ is presented in Table I. The table shows that the responses with STLCDs and MTLCDs are nearly identical. Reductions of up to 47 per cent in the relative displacements and absolute accelerations are observed when TLCDs are used.

Ten-storey building modelled as MDOF structure

A ten-storey building with an assumed damping ratio of 0·02 in the first mode is to be designed for a peak ground acceleration of $0{\cdot}4g$. The assumed storey masses and column stiffnesses from the top to bottom are: $\{98, 107, 116, 125, 134, 143, 152, 161, 170, 179\} \times 10^3$ kg and $\{34{\cdot}31, 37{\cdot}43, 40{\cdot}55, 43{\cdot}67, 46{\cdot}79, 49{\cdot}91, 53{\cdot}02, 56{\cdot}14, 59{\cdot}26, 62{\cdot}47\} \times 10^3$ kN/m, respectively. The building has a fundamental natural frequency of 0·5 Hz. Two cases, one with STLCD and another with MTLCD attached to the top floor are considered. The selection of parameters is the same as before except that the mass ratio is computed as the ratio of the liquid mass to the generalized mass for the fundamental mode with a unit modal participation factor, i.e.

$$\mu = \frac{\rho A L}{\phi_1^T [M] \phi_1} \tag{17}$$

where $[M]$ is the structure mass matrix and ϕ_1 the fundamental mode shape normalized to have a unit participation factor. For the structure considered, the generalized mass for the fundamental mode is 1109×10^3 kg. If the STLCD and MTLCD are designed for a mass ratio of 0·04, the liquid mass would be $44{\cdot}36 \times 10^3$ kg which is equal to approximately 0·03 of the total structural mass and 0·25 of the first floor mass. For the STLCD, 800 U-tubes, each with a liquid length of 2·2 m and a cross-sectional area of 0·025 m^2 may be used. For the MTLCD, five groups, each with 175 U-tubes would be selected. Each U-tube would have a cross-sectional area of 0·025 m^2 and liquid lengths of 1·75, 1·86, 1·99, 2·13, and 2·27 m for groups 1–5, respectively. The peak ground acceleration of $0{\cdot}4g$ results in a head loss coefficient $\delta = 0{\cdot}358$, equation (14). The building with and without TLCDs was subjected to the 90° component of the Corralitos Eureka Canyon Road accelerogram and the 90° component of the Capitola Fire Station accelerogram from the Loma Prieta earthquake of 17 October 1989; and the 90° component of the Santa Monica City Hall Grounds accelerogram and the 90° component of the Arleta Nordhoff Avenue Fire Station accelerogram from the Northridge earthquake of 17 January 1994; each scaled to a peak ground acceleration of $0{\cdot}4g$. The results of the analyses, summarized in Table II, show a reduction of up to 40 per cent in the displacement and 24 per cent in the acceleration of the top floor. Both STLCD and MTLCD result in approximately the same response reduction.

This structure was also analysed with a tuned mass damper attached to the top floor. It is assumed that the TMD has the same mass ratio as those of the STLCD and MTLCD ($\mu = 0{\cdot}04$). The method presented by Sadek *et al.*[14] was used to select the TMD parameters. The tuning ratio of the TMD is 0·994 and the damping

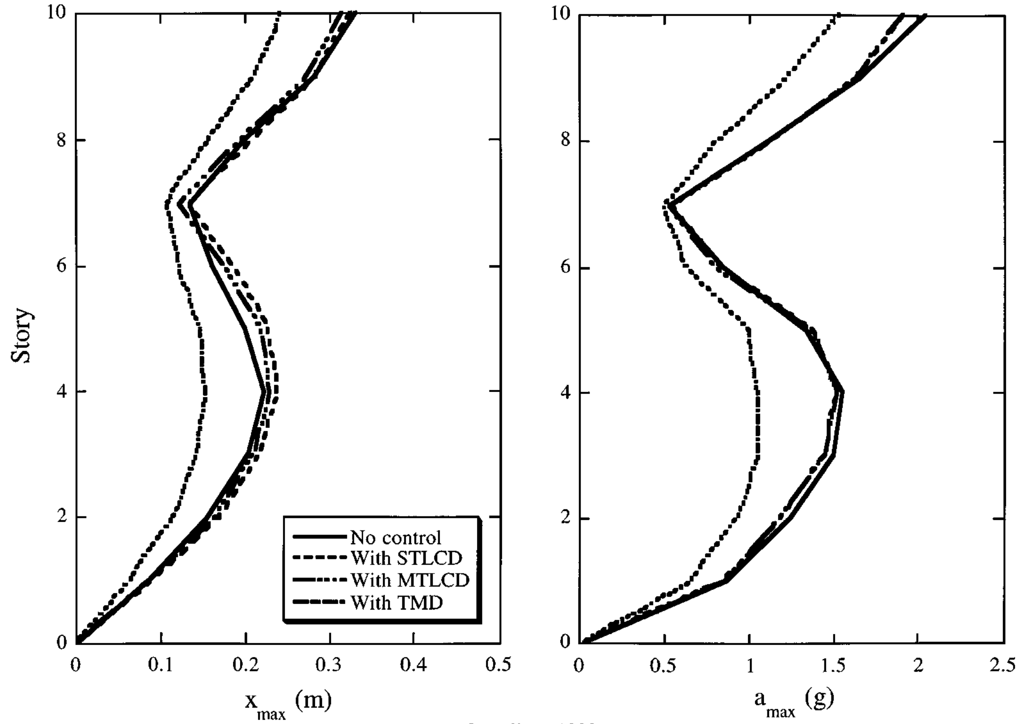
Table II. Response of the ten-storey building with and without TLCD

	Corralitos, 1989						Capitola, 1989					
	No control		STLCD		MTLCD		No control		STLCD		MTLCD	
Level	x_{max} m	a_{max} g	x_{max} m	a_{max} g	x_{max} m	a_{max} g	x_{max} m	a_{max} g	x_{max} m	a_{max} g	x_{max} m	a_{max} g
Top	0·331	2·04	0·325	1·91	0·314	1·91	0·258	2·05	0·156	1·63	0·163	1·63
9	0·282	1·66	0·283	1·63	0·272	1·63	0·215	1·12	0·133	1·03	0·137	0·98
8	0·200	1·12	0·208	1·13	0·195	1·12	0·205	1·40	0·103	1·07	0·103	1·12
7	0·136	0·53	0·134	0·55	0·121	0·53	0·200	1·44	0·114	1·11	0·112	1·12
6	0·162	0·85	0·186	0·84	0·175	0·81	0·182	1·06	0·123	1·06	0·121	1·07
5	0·200	1·34	0·228	1·38	0·219	1·37	0·160	1·33	0·125	1·08	0·123	1·12
4	0·223	1·55	0·238	1·52	0·230	1·52	0·150	1·37	0·106	1·11	0·106	1·08
3	0·204	1·50	0·217	1·45	0·210	1·45	0·137	1·04	0·084	0·94	0·084	0·94
2	0·155	1·25	0·169	1·18	0·164	1·18	0·122	1·44	0·072	1·32	0·074	1·33
1	0·086	0·87	0·091	0·85	0·089	0·86	0·072	1·30	0·050	1·07	0·049	1·07
	Santa Monica, 1994						Arleta, 1994					
Top	0·219	1·04	0·217	0·92	0·217	0·92	0·463	1·90	0·309	1·44	0·279	1·43
9	0·201	0·71	0·198	0·71	0·189	0·70	0·443	1·20	0·292	1·21	0·252	1·18
8	0·182	0·81	0·179	0·69	0·178	0·67	0·414	1·38	0·262	0·97	0·214	0·98
7	0·170	0·66	0·162	0·68	0·171	0·67	0·356	1·00	0·215	0·87	0·184	0·87
6	0·164	0·64	0·153	0·69	0·160	0·65	0·338	1·03	0·162	0·87	0·168	0·92
5	0·146	0·81	0·130	0·66	0·141	0·67	0·311	1·40	0·160	1·20	0·174	1·20
4	0·129	0·69	0·107	0·67	0·115	0·67	0·275	1·32	0·163	0·99	0·166	1·01
3	0·103	0·63	0·085	0·65	0·090	0·64	0·218	1·02	0·148	1·11	0·144	1·09
2	0·074	0·59	0·063	0·58	0·068	0·59	0·154	1·13	0·116	1·16	0·102	1·14
1	0·042	0·72	0·039	0·70	0·039	0·70	0·083	1·05	0·064	0·93	0·058	0·93

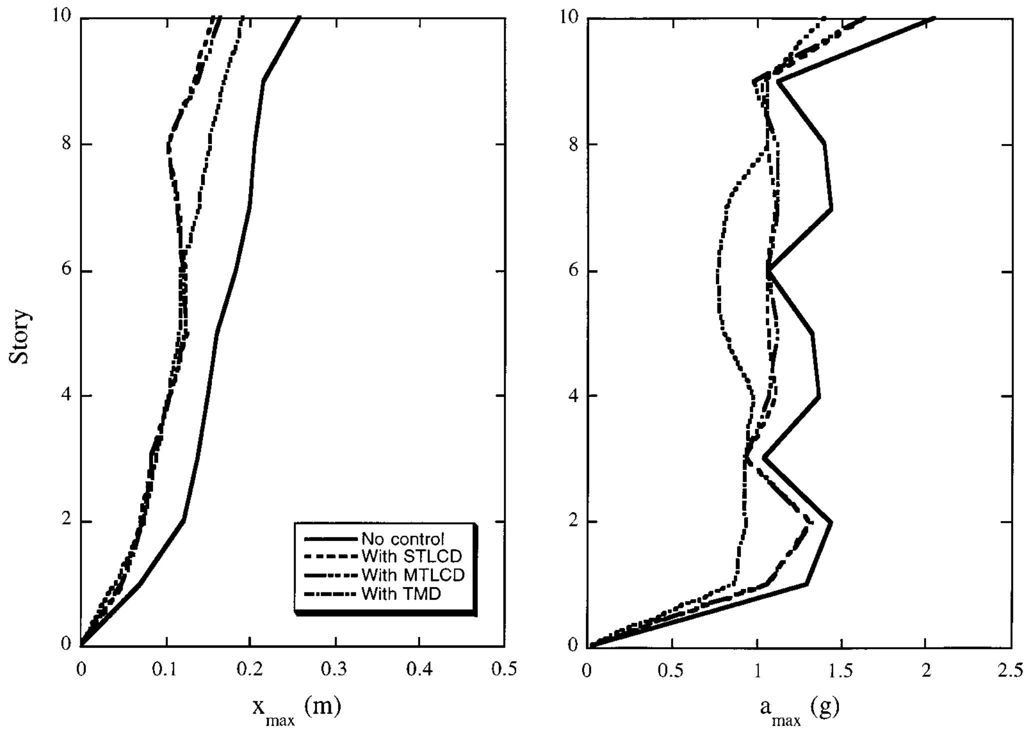
ratio 0·293. The responses of the structure with no control and with STLCD, MTLCD and TMD to the four ground excitations are plotted in Figure 14. The plots show that the performance of both TMDs and TLCDs are comparable. Table II and Figure 14 show that for some records (Capitola and Arleta), both devices result in significant reductions in the response while for other records (Corralitos and Santa Monica), the reductions are not as significant.

The reason that reductions in the response are observed for some records but not others is the peaks and valleys in the response spectrum of the records. The addition of the STLCD introduces one more degree of freedom and shifts the fundamental frequency of 0·5 Hz to frequencies of 0·42 and 0·56 Hz. An examination of the response spectra for Capitola and Arleta accelerograms (Figure 15(a)) shows that the responses at 0·42 and 0·56 Hz (structure with TLCD) are smaller than the response at 0·5 Hz (structure without TLCD). This is not the case, however, with the response spectra for Corralitos and Santa Monica (Figure 15(b)) where the response at 0·5 Hz is between the responses at 0·42 and 0·56 Hz. Therefore, the addition of TLCD was not as effective.

The above observation underscores that the performance of TMDs and TLCDs is influenced by the frequency of the structure and the frequency content of the excitation which is characteristic of all passive systems. It should be noted that a similar behaviour is observed for structures without any control where peak and valleys in a response spectrum may influence the response to a substantial degree.

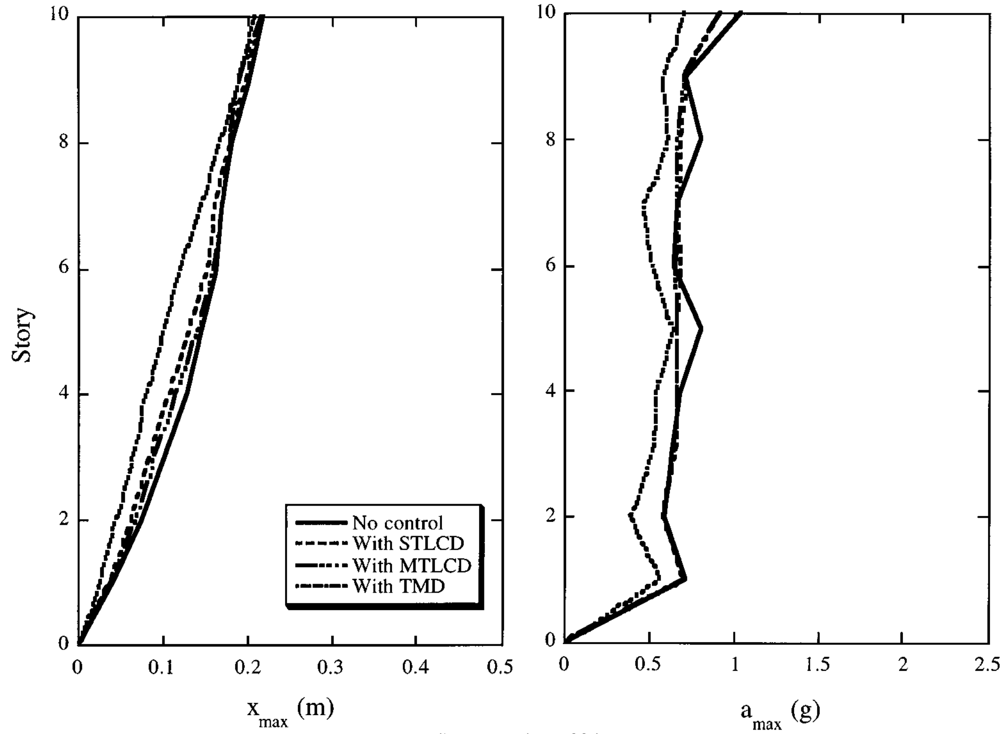


(a) Corralitos, 1989

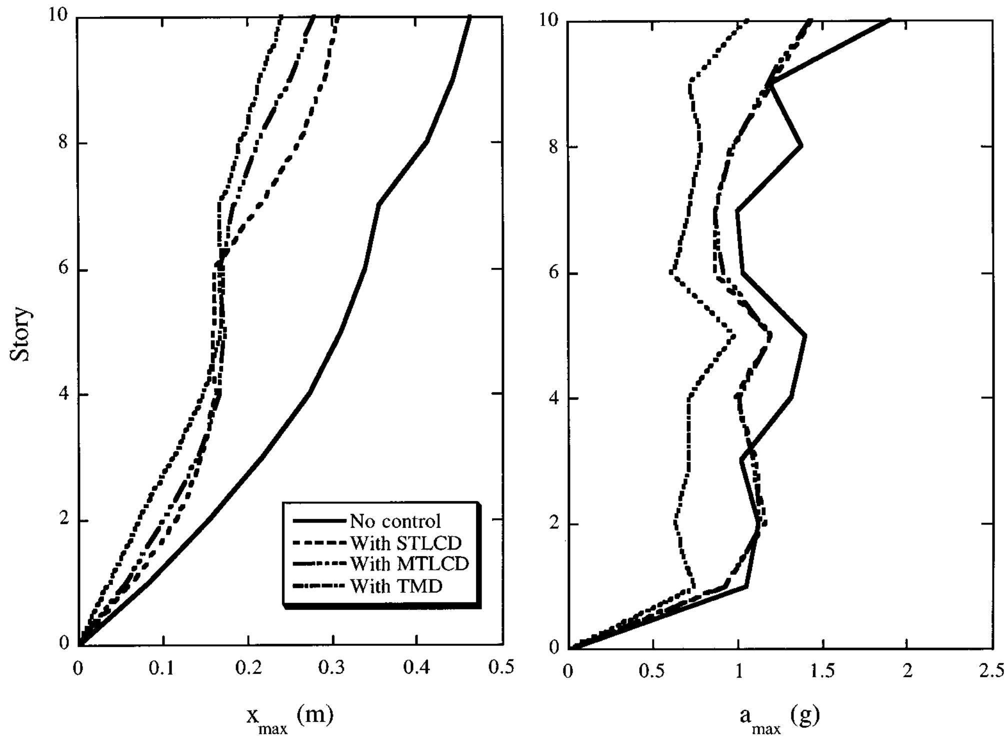


(b) Capitola, 1989

Figure 14. Peak responses of a ten-storey building with no control and with STLCD, MTLCD, and TMD to four ground excitations



(c) Santa Monica, 1994



(d) Arleta, 1994

Figure 14. (*Continued*)

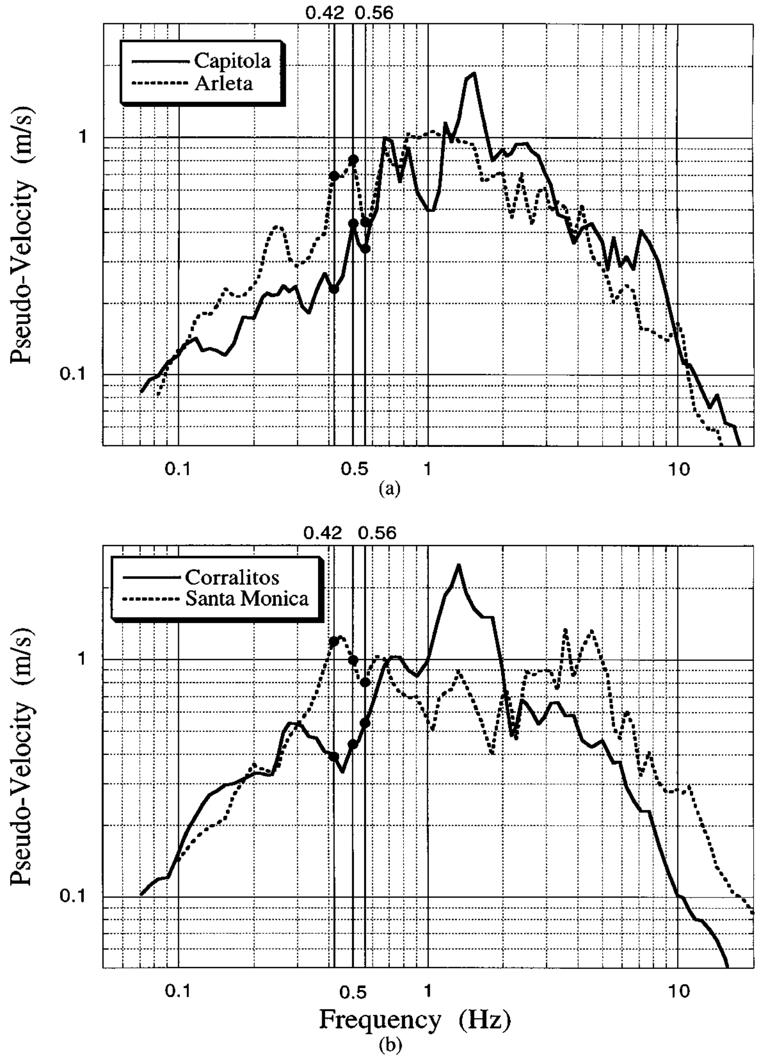


Figure 15. Response spectra for a structural damping ratio of 0·02 showing the effect of frequency shift caused by the addition of STLCD

CONCLUSIONS

The objective of this study was to determine the design parameters for tuned liquid column dampers (TLCDs) for seismic applications. The design parameters for single-tuned liquid column dampers (STLCDs): tuning, damping, and liquid length to tube width ratios; and for multiple-tuned liquid column dampers (MTLCDs): central tuning ratio, tuning bandwidth, and number of TLCD groups are determined from a deterministic response analysis of SDOF structures to 72 earthquake accelerograms. The parameters were used to compute the response of several single-degree-of-freedom and multi-degree-of-freedom structures with single and multiple TLCDs to different earthquake excitations. The results indicate that selecting the parameters as described in this paper results in displacement and acceleration response reductions of up to 47 per cent. The study shows that while multiple-tuned liquid column dampers are not necessarily superior to

single-tuned liquid column dampers, they are robust with respect to errors in estimating the structural parameters. Comparisons with tuned mass dampers indicate that both devices are comparable in reducing the response of structures. Design examples for STLCD and MTLCD used in a bridge modelled as a SDOF structure and a ten-storey building modelled as a MDOF structure are presented to illustrate the selection of the parameters and demonstrate the performance of the STLCDs and MTLCDs under different ground excitations. The performance of tuned liquid column dampers was also compared with that of tuned mass dampers where it was found that both devices result in similar reductions in the response. These reductions, however, are influenced by the frequency content of the excitation.

APPENDIX I

List of symbols

a	peak ground acceleration
$a_{\max}$	maximum absolute acceleration
A	cross-sectional area of the liquid damper
B	tube width
c_p	equivalent damping coefficient of TLCD
f	tuning ratio of STLCD
f_0	central frequency ratio of MTLCD
g	acceleration due to gravity
L	liquid column length
M	mass of an SDOF structure or the generalized mass in an MDOF structure
$[M]$	mass matrix
N	numbers of groups in a MTLCD
T	natural period
v	peak ground velocity
x	displacement of the main structure
$\ddot{x}_g$	ground acceleration
$x_{\max}$	maximum relative displacement
y	elevation change of the liquid surface
α	tube width to liquid length ratio
β	damping ratio of structure
δ	coefficient of head loss of the damper
Δf	frequency bandwidth of MTLCD
η	constant that depends on μ and β
ϕ_1	fundamental modal shape
μ	mass ratio of TLCD
ρ	liquid density
$\sigma_{\dot{y}}$	standard deviation of the liquid elevation velocity
ω_0	natural or fundamental frequency of the structure
ω_t	natural frequency of TLCD
ξ	equivalent damping ratio of TLCD

APPENDIX II

Earthquake records used in the statistical study

Earthquake	Mag.	Station name	Source distance (km)	Comp.	Peak accel. (g)
Imperial Valley	6·7	El Centro Valley	11·6	S00E	0·348
05/18/1940		Irrigation District		S90W	0·214
Northwest California	5·8	Ferndale City Hall	56·3	S44W	0·104
10/07/1951				N46W	0·112
Kern County	7·7	Pasadena-Caltech	127·0	S00E	0·047
06/21/1952		Athenaeum		S90W	0·053
		Taft Lincoln School	41·4	N21E	0·156
		Tunnel		S69E	0·179
		Santa Barbara Court	88·4	N42E	0·089
		House		S48E	0·131
		Holywood Storage	120·4	S00W	0·055
		Basement		N90E	0·044
Eureka	6·5	Ferndale City Hall	40·0	N44E	0·159
12/21/1954				N46W	0·201
San Francisco	5·3	San Francisco Golden	11·2	N10E	0·083
03/22/1957		Gate Park		S80E	0·105
Hollister	5·7	Hollister City Hall	22·1	S01W	0·065
04/08/1961				N89W	0·179
Borrego Mountain	6·4	El Centro Valley	67·3	S00W	0·130
04/08/1968		Irrigation District		S90W	0·057
Long Beach	6·3	Vernon CMD Bldg.	50·5	S08W	0·133
03/10/1933				N82W	0·155
Lower California		El Centro Valley	66·4	S00W	0·160
12/30/1934	7·1	Irrigation District		S90W	0·182
Helena Montana	6·0	Helena, Montana	6·2	S00W	0·146
10/31/1935		Carrol College		S90W	0·145
1st Northwest California	5·5	Ferndale City Hall	55·2	N45E	0·144
09/11/1938				S45E	0·089
Northern California	5·2	Ferndale City Hall	43·1	N44E	0·054
09/22/1952				S46E	0·076
Wheeler Ridge, California	5·9	Taft Lincoln School	42·8	N21E	0·065
01/12/1954		Tunnel		S69E	0·068
Parkfield, California	5·6	Chalome, Shandon,	56·1	N05W	0·355
06/27/1966		California Array #5		N85E	0·434
		Cholame, Shandon,	53·6	N50E	0·053
		California Array #12		N40W	0·064
		Temblor, California	59·6	N65W	0·269
		#2		S25W	0·347

APPENDIX II (*Continued*)

Earthquake records used in the statistical study

Earthquake	Mag.	Station name	Source distance (km)	Comp.	Peak accel. (g)
San Fernando 02/09/1971	6·4	Pacoima Dam	7·3	S16E S74W	1·172 1·070
		8244 Orion Blvd. Los Angeles, California	21·1	N00W S90W	0·255 0·134
		250 E First Street Basement, Los Angeles	41·4	N36W N54W	0·100 0·125
		Castaic Old Ridge Route	29·5	N21E N69W	0·315 0·270
		7080 Hollywood Blvd. Basement, Los Angeles	33·5	N00E N90W	0·083 0·100
		Vernon CMD Bldg.	48·0	N83W S07W	0·107 0·082
		Caltech Seismological Lab., Pasadena	34·6	S00W S90W	0·089 0·193
Loma Prieta 10/17/1989	7·1	Corralitos-Eureka Canyon Road	7·0	90° 0°	0·478 0·630
		Capitola-Fire Station	9·0	90° 0°	0·398 0·472
		Foster City-Redwood Shores	63·0	90° 0°	0·283 0·258
		Monterey-City Hall	49·0	90° 0°	0·062 0·070
		Woodside-Fire Station	55·0	90° 0°	0·081 0·081
Northridge 01/17/1994	6·7	Arleta Nordhoff Ave.-Fire Station	9·9	90° 360°	0·344 0·308
		New Hall-LA County Fire Station	19·8	90° 360°	0·583 0·589
		Pacoima Dam-Down Stream	19·3	265° 175°	0·434 0·415
		Santa Monica-City Hall Grounds	22·5	90° 360°	0·883 0·370
		Sylmar-County Hospital Parking Lot	15·8	90° 360°	0·604 0·843

ACKNOWLEDGMENTS

This study was supported by the Structures Division, Building and Fire Research Laboratory, National Institute of Standards and Technology, U.S. Department of Commerce through a grant to the Mechanical Engineering Department, Southern Methodist University, Dallas, Texas.

REFERENCES

1. K. C. S. Kwok, Y. L. Xu and B. Samali, 'Control of wind-induced vibrations of tall structures by optimized tuned liquid column dampers', in Y. K. Cheung, J. H. W. Lee and A. Y. T. Leung (eds), *Computational Mechanics*, Balkena, Rotterdam, 1991, pp. 249–254.
2. Y. L. Xu, B. Samali and K. C. S. Kwok, 'Control of along-wind response of structures by mass and liquid dampers', *J. Eng. Mech. ASCE* **118**, 20–39 (1992).
3. Y. Fujino and L. M. Sun, 'Vibration control by multiple tuned liquid dampers (MTLDs)', *J. Engng. Mech. ASCE* **119**, 3482–3502 (1993).
4. A. Kareem, 'The next generation of tuned liquid dampers', *Proc. 1st World Conference on Structural Control*, 3–5 August, Los Angeles, CA, USA, 1994, pp. FP5-19-28.
5. F. Sakai, S. Takaeda and T. Tamaki, 'Tuned liquid column dampers (TLCD) for cable-stayed bridges', *Proc. Speciality Conf. Innovation in Cable-Stayed Bridges*, Fukuoka, Japan, 1991, pp. 197–205.
6. M. A. Haroun, J. A. Pires and A. Y. J. Won, 'Hybrid liquid column dampers for suppression of environmentally-induced vibrations in tall buildings', *Proc. 3rd Conf. Tall Buildings in Seismic Regions*, Los Angeles, CA, 1994.
7. M. A. Haroun, J. A. Pires and A. Y. J. Won, 'Active orifice control in hybrid liquid column dampers', *Proc. 1st World Conf. on Structural Control*, 3–5 August, Los Angeles, CA, USA, 1994, pp. FA1-69-78.
8. F. Sakai, S. Takaeda and T. Tamaki, 'Tuned liquid column dampers—new type device for suppression of building vibrations', *Proc. Int. Conf. on Highrise Buildings*, Vol. 2, Nanjing, China, 1989, pp. 926–931.
9. R. D. Blevins, *Applied Fluids Dynamics Handbook*, Von Nostrand Reinhold, New York, 1984.
10. K. Sun, 'Earthquake response of buildings with liquid column dampers', *Proc. 5th U.S. National Conf. on Earthquake Engineering*, 10–14 July, Chicago, IL, Vol. II, 1994, pp. 411–420.
11. A. Y. J. Won, J. A. Pires and M. A. Haroun, 'Stochastic seismic performance evaluation of tuned liquid column dampers', *Earthquake Engng Struct. Dyn.* **25**, 1259–1274 (1996).
12. G. B. Warburton, 'Optimum absorber parameters for various combinations of response and excitation parameters', *Earthquake Engng Struct. Dyn.* **10**, 381–401 (1982).
13. R. Villaverde, 'Reduction in seismic response with heavily-damped vibration absorbers', *Earthquake Engng. Struct. Dyn.* **13** (1985) 33–42.
14. F. Sadek, B. Mohraz, A. W. Taylor and R. M. Chung, 'A method of estimating the parameters of tuned mass dampers for seismic applications', *Earthquake Engng. Struct. Dyn.* **26**, 617–635 (1997).
15. H. Yamaguchi and N. Harnpornchai, 'Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations', *Earthquake Engng. Struct. Dyn.* **22**, 51–62 (1993).